CENTRAL AND SOUTH-EASTERN EUROPEAN STOCK MARKETS: EVIDENCE OF LONG-MEMORY IN MEAN AND VOLATILITY

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Abstract: *The paper examines the presence of long-memory in mean and volatility of five capital markets returns from Central and South-Eastern Europe (Hungary, the Czech Republic, Bulgaria, Croatia and Serbia). Daily stock market indices are considered covering period 2001-2012, except that for Serbia sample starts in 2005. The results indicate that long-memory is not a relevant property of returns on BUX (Hungary) and PX (the Czech Republic). However, long-range dependence has been estimated as highly significant in returns of SOFIX (Bulgaria), CROBEX (Croatia) and BELEX15 (Serbia). Thus, ARFIMA specification was chosen as appropriate modelling framework. For all five time series long-memory was found to be the key characteristic of conditional variability as well. FIGARCH set-up was found to explain volatility satisfactory well. In addition, significant asymmetric behaviour of fractionally-differencing volatility has been revealed for BUX and PX returns.*

The strong evidence of long-memory in Bulgarian, Croatian and Serbian stock returns shows that weak-form market efficiency cannot be associated with these data. Only financial markets in Hungary and Czech Republic have exhibited some improvements towards reaching efficiency. The presence of long-memory in volatility of all five returns is a sign of its high persistence that should not be neglected when risk is to be estimated.

Key words: *Long-memory, ARFIMA models, FIGARCH models, stock market return, market efficiency.*

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1. Introduction

The presence of long-memory in returns suggests that distant observations are significantly correlated. Economically interpreted, new information is not immediately digested by the market, but, instead, market reacts to it with a certain lag. This rejects weak-form market efficiency and opens the possibility of systematically gaining speculative profit. Given the proper detection of long-memory, a profitable trading strategy could be developed.

Long-memory can be found in volatility of returns as well. It implies wide-distant correlation of time-varying volatility components. Therefore, volatility moves non-randomly exhibiting long-range regularity pattern. We may argue that such a dynamics makes uncertainty or risk associated to price and return movements highly persistent.

Emerging capital markets have features that would suggest long-memory to be expected. These markets are attractive for investors, which is primarily due to their low correlation with developed markets making them a significant source of portfolio diversification. However, investors tend to react slowly and unpredictable in these markets that are commonly characterized as being thin, highly volatile and with nonsyncronous trading. In addition, regulatory framework changes often (Hull and McGroarty 2014).

The presence of long-memory in mean or (and) volatility of stock returns have been widely considered for many emerging markets. Empirical evidence is mixed, but with the prevailing results that confirm the relevance of long-memory property. Wright (2001) is one of the first papers that consider this issue across several emerging markets. Many individual emerging market countries or a subset of them have also been under investigation. Selected list of references includes: Sadique and Silvapulle 2001; Henry 2002; Sourial 2002; Limam 2003; Kilic 2004; Gil-Alana 2006; Assaf 2006; Kasman and Torun 2007; Kang and Yoon 2007; Kasman et al. 2009; Hiremath and Bandi 2011; Bhattacharya and Bhattacharya 2012; Hull and McGroarty 2014.

Emerging capital markets of Central and South-Eastern Europe (CSEE) have several peculiar characteristics. First of all, these countries went through massive and deep economic and political reforms during the period of transition towards market economies in the 1990s. Their capital markets started to operate or re-operate at the beginning of that period. Second, most of these markets have been characterized by high volatility due to both, high sensitivity to changes in regional and global adjustments of large investments funds and low level of liquidity (Kasman et al. 2009). Nevertheless, capital markets in some of the countries (Hungary and Czech Republic) have reached substantially high level of development in respect to market capitalization, daily trade volumes and integration with world financial institutions. The dual long-memory properties of returns from eight CSEE countries have been extensively investigated by Kasman et al. 2009. However, most of these results do not cover recent period of financial turmoil that started in the second half of 2008.

The purpose of this paper is to exploit potential presence of long-memory in the level and volatility of several capital markets from CSEE region. Our sample includes countries at different level of capital market developments with Hungary and Czech Republic at one end and Serbia, being a late comer to transition process, at the other end. Croatia and Bulgaria are also considered. Daily data are used for the sample that encompasses post 2008 period, thus enabling assessment on recent behavior of these markets. The question is whether they have shown improvement towards more efficiency despite the 2008 financial crisis. Our empirical analysis employs a two-pass approach (Tsay 2010) that takes into account step by step long-memory in mean and volatility of returns. For that purpose ARFIMA and FIGARCH models are applied as key methodological framework.

Paper is structured as follows. Basic time series features are covered in Section 2. The most important methodological issues are overviewed by Section 3, while Section 4 contains results on testing for the dual long-range dependence. Estimated models are presented and discussed in Section 5. Conclusions are summarized in Section 6.

2. Basic Data Analysis

The sample of the research comprises daily returns of stock indices of selected Central and South-Eastern European countries that are: Hungary, Czech Republic, Croatia, Bulgaria and Serbia. The considered stock indices are: BUX, PX, CROBEX, SOFIX and BELEX15 respectively. The first three indices are analyzed during the period January, 2001 – December, 2012, for the fourth one sample starts in January, 2002, while the data for the last stock index cover the sample October, 2005 – December, 2012, in respect. The data are obtained from national stock exchange websites. For each index, P_t, we compute daily logarithmic return as: $(\ln P_t - \ln P_{t-1})*100$. The sample of this structure and time dimension has not been discussed in empirical literature. Empirical results are derived from Oxmetrics software (Doornik 2009).

To provide first insight into the data properties each return is depicted. Sample autocorrelation correlogram for the first 20 lags is given along with the histogram and Q-Q plot against the normal distribution. Graphs are contained in Figures 1 – 5.

Figure 1: BUX return

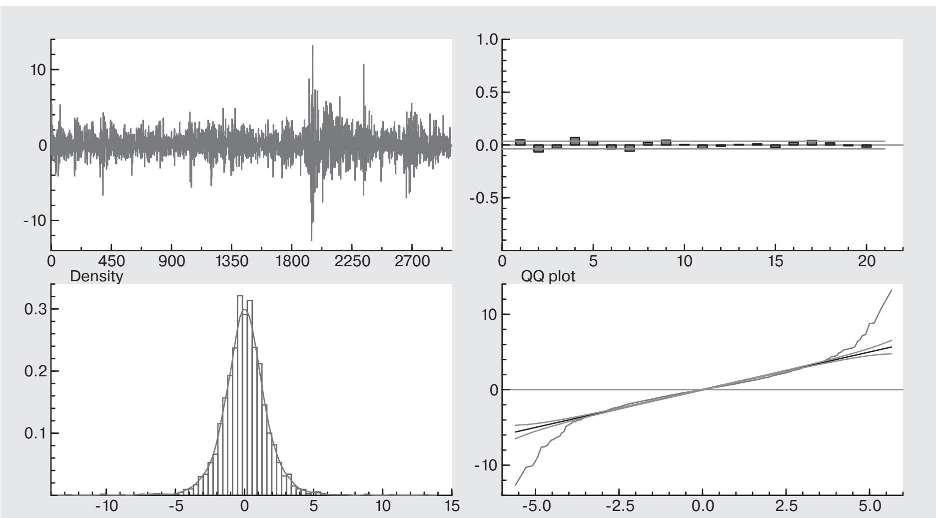


Figure 2: PX return

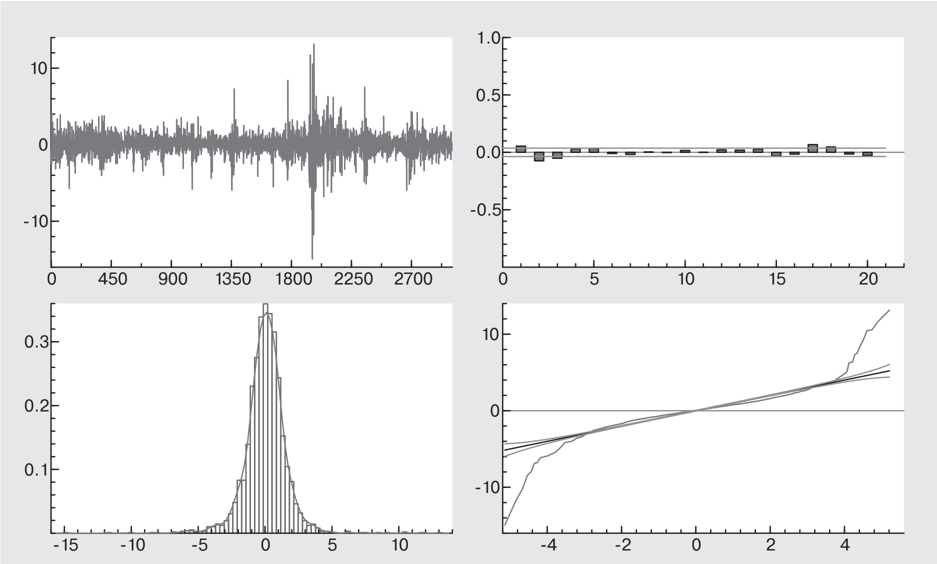


Figure 3: SOFIX return

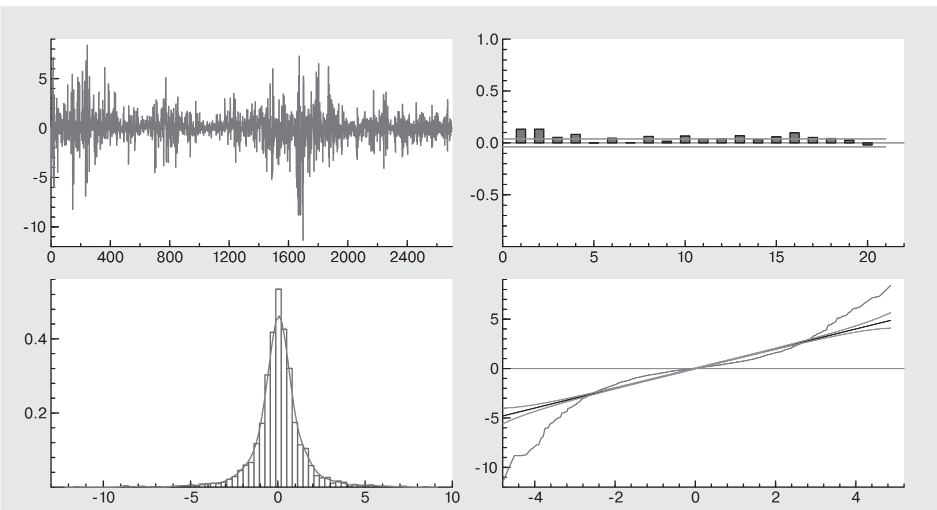


Figure 4: CROBEX return

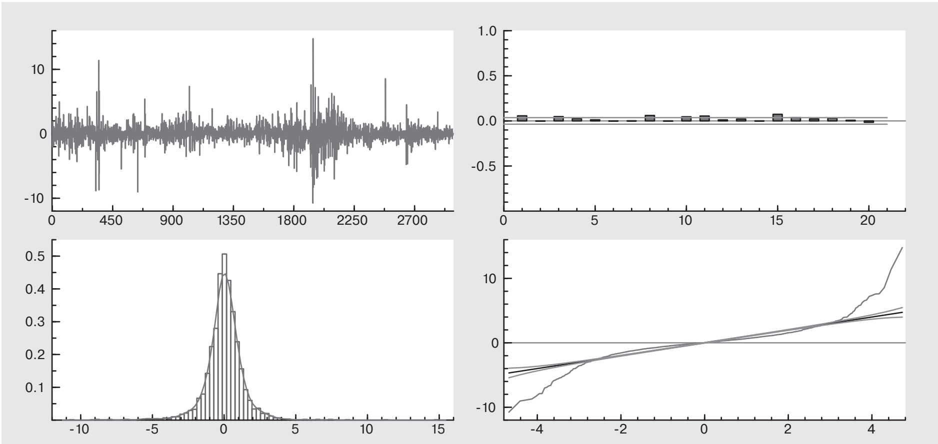
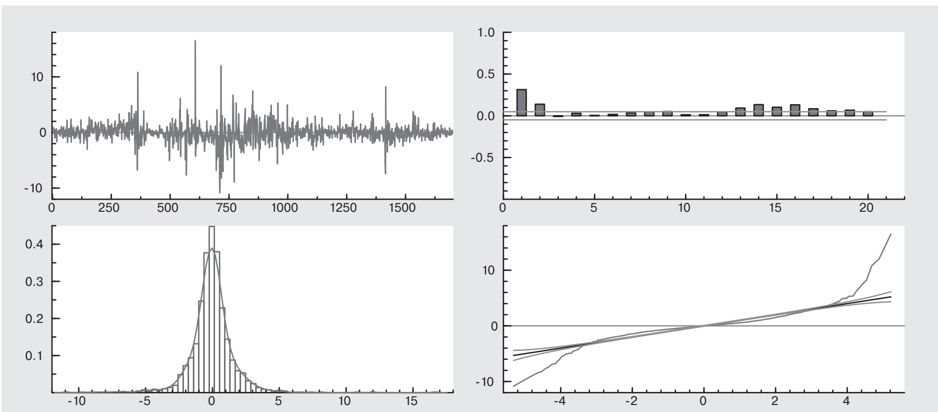


Figure 5: BELEX15 return



Volatility clustering is clearly visible from the data graph. Empirical distributions depart substantially from the normal distribution as pointed out by histogram and Q-Q plots. All data exhibit significant autocorrelation.

More information on data properties are given in Table 1. It indicates that the daily returns of all five market stock indices are not normally distributed. Skewness is evident in three cases. Excess kurtosis is significant in all cases as it is the Jarque-Bera statistics. The results imply the presence of fat tails, which questions the assumption of a normal distribution in empirical modelling. ARCH-LM and Box-Ljung Q^2 test point to the significant autoregressive structure of volatility and thus relevance of using ARFIMA models.

Table 1: Descriptive characteristics of stock exchange daily returns

Index	Skewness	Excess Kurtosis	JB	Q^2 (20)	ARCH-LM (10)
BUX	-0.08 (0.07)	5.91 (0.00)	4353 (0.00)	2575.8 (0.00)	88.9 (0.00)
PX	-0.18 (0.00)	11.85 (0.00)	17586 (0.00)	3879.9 (0.00)	120.3 (0.00)
BELEX15	0.42 (0.00)	14.27 (0.00)	14465 (0.00)	340.74 (0.00)	17.38 (0.00)
CROBEX	-0.01 (0.81)	13.56 (0.00)	22856 (0.00)	1852.14 (0.00)	87.63 (0.00)
SOFIX	-0.51 (0.00)	7.96 (0.00)	7243 (0.00)	2066.6 (0.00)	79.54 (0.00)

Note: p values of corresponding test statistics are given in parentheses. JB represents the Jarque-Bera statistics for normality testing, Q^2 represents the Box-Ljung statistics for testing autocorrelation of order 20 in squared data, while ARCH-LM test examines the presence of autoregressive conditional heteroskedasticity of order 10 in the form of F statistic.

3. Methodology Overview

This section shortly overviews models commonly applied to capture the presence of long-memory in the level and volatility. Some of the tests used to detect the long-rang dependence are also covered.

Autocorrelation function (ACF), as already employed, is a standard tool in discovering statistical properties of time series. For most stationary time series ACF decays exponentially to zero as lag increases. There are, however, stationary time series characterized by ACF that decays slowly to zero at a polynomial rate with the lag increase. Such time series are known as long-memory time series or long-range dependent time series. A typical specification used to capture such a behavior is fractionally differenced time series model (Granger and Joyeux 1980; Hosking, 1981):

$$(1-L)^{\mathrm{d}} x_t = e_t, e_t : iid\left(0, \sigma_e^2\right) \tag{1}$$

where: L is the lag-one operator and e_t is an error term. A key parameter of the model, d, is parameter of fractional differencing. It should satisfy the condition: –0.5<d<0.5 for the time series to be stationary and invertible. The associated ACF behaves as (Gil-Alana and Hualde 2009):

$$\frac{(-\mathrm{d})!}{(\mathrm{d}-1)!} k^{2\mathrm{d}-1}, k \rightarrow \infty \tag{2}$$

For 0<d<0.5 ACF takes positive values that decay to zero at rate $k^{2\mathrm{d}-1}$ which represents hyperbolic pattern. Such a process is called a long-memory time series (Tsay 2010).

If the fractionally differenced time series, $(1–L)^d x_t$, follows an ARMA(p,q) structure, then time series of interest, x_t, is described by ARFIMA(p,d,q) specification.

The fractional differencing parameter can be estimated by several methods. One of the first approaches defined is based on the log-periodogram regression suggested by Geweke and and Porter-Hudak 1983 (GPH estimate). Parameter d can also be estimated by the method of maximum likelihood (Beran 1985).

Long-memory may appear in conditional variance. As a flexible class of model capable of explaining and representing the temporal dependencies in financial market volatility the fractionally integrated GARCH model (FIGARCH) has been defined by Baillie 1996 and Baillie, Bollerslev and Mikkelsen 1996. The volatility of traditional GARCH(m,s) model (σ_t^2) is given by (Bollerslev 1986):

$$\sigma_t^2 = \alpha_0 + \alpha(L)e_t^2 + \beta(L)\sigma_t^2, e_t = z_t\sigma_t, z_t : N(0,1) \tag{3}$$

which can be rewritten as ARMA specification for the squared error term (e_t^2):

$$(1 - \alpha(L) - \beta(L))e_t^2 = \alpha_0 + (1 - \beta(L))v_t, v_t = e_t^2 - \sigma_t^2. \tag{4}$$

α(L) and β(L) are polynomials in lag operator L of order s and m respectively. Their parameters are given respectively by $\alpha_1,\ldots,\alpha_s$ and $\beta_1,\ldots,\beta_m$. Constant term is denoted by α_0.

The FIGARCH form assumes that squared error term needs fractional differencing prior to modeling. Fractional differencing parameter is denoted by d and it takes values between 0 and 1:

$$\underbrace{(1-\alpha(L)-\beta(L))}_{\Phi(L)}(1-L)^d e_t^2 = \alpha_0 + (1-\beta(L))v_t \tag{5}$$

The general form of the model is given by FIGARCH(m,d,s). Newly defined polynomial in lag operator L, Φ(L), has parameters ϕ_1, ϕ_2, etc. Alternative representations of (5) are:

$$(1 - \beta(L))\sigma_t^2 = \alpha_0 + (1 - \beta(L) - \Phi(L)(1 - L)^d)e_t^2 \tag{6}$$

or

$$\sigma_t^2 = \frac{\alpha_0}{(1 - \beta(L))} + \left(1 - \frac{\Phi(L)(1 - L)^d}{(1 - \beta(L))}\right)e_t^2 \tag{7}$$

Necessary and sufficient conditions for the positivity of conditional variance are different than in GARCH models. For example, conditional variance from FIGARCH ($1,d,1$) model is positive under the following restrictions (Conrad and Haag 2006): $\alpha_0 > 0, (\alpha_1 + d) > 0,\ 1 - 2(\alpha_1 + \beta_1) \geq d \geq 0.$

Several modifications of FIGARCH form are proposed in the literature. To incorporate asymmetric reaction of volatility to positive and negative shocks the asymmetric power FIGARCH model is defined (AP-FIGARCH). It has the following form (Tse 1998; Doornik 2009):

$$\sigma_t^{\delta} = \frac{\alpha_0}{(1-\beta(L))} + \left(1 - \frac{\Phi(L)(1-L)^d}{(1-\beta(L))}\right)(|e_t| - \gamma_1 e_t)^{\delta},\ \delta > 0, |\gamma_1| < 1 \tag{8}$$

Parameter δ denotes power of standard deviation, while parameter γ_1 captures different impact on volatility depending on the sign of shock in previous periods.

Parameters of different FIGARCH specifications can be estimated by the method of maximum likelihood (Bollerslev and Wooldridge 1992). As the assumption that the error term follows normal distribution is often not plausible for economic data, it is possible to perform estimation under the assumption that the error term has t-distribution. Number of degrees of freedom of t-distribution (v) is estimated together with other parameters of the model.

To formally test for the presence of long-term memory in time series rescaled range statistic R/S defined by Hurst 1951 and Mandelbrot 1972 is often applied. Basically it represents the range of partial sums of deviations of a time series from its mean that are rescaled by its standard deviation:

$$R/S = \hat{\sigma}_0^{-1}\left[\max_{1\le i\le T}\sum_{t=1}^{i}(x_t - \bar{x}) - \min_{1\le i\le T}\sum_{t=1}^{i}(x_t - \bar{x})\right], \hat{\sigma}_0^2 = T^{-1}\sum_{t=1}^{T}(x_t - \bar{x})^2 \tag{9}$$

This statistic is robust to data non-normality, but its result may depend on data short-run variations. To account for this short-term dependence Lo 1991 suggested another type of R/S statistic in which standard deviation $\hat{\sigma}_0$ is obtained by the Newey-West modification. Distributions of both statistics converge under certain assumptions to the range of a Brownian bridge on the unit interval as the sample size and lag-window of the Newey-West correction increase.

Another test defined for the long-memory detection is the runs test of Fama 1965. It is designed as a non-parametric test that examines the randomness hypothesis of a two-valued data sequence. First we introduce $RT = 1 + \sum_{t=1}^{T-1} k_t$ where k_t is equal to 1 if the return at time t has the same sign (positive, negative or zero) as the return at time t+1. Otherwise, the value of k_t is 0. The runs test is calculated as:

$$z = (RT - E(RT)) / \sqrt{V(RT)} \tag{10}$$

where expected value, *E(RT),* and variance, *V(RT),* are defined based on the total number of positive, negative and zero returns. Asymptotic distribution of the runs test is standard normal (Doornik 2009).

The application of ARFIMA and FIGARCH models in micro and macroeconomic researches is vast (see Gil-Alana and Hualde 2009). For example, inflation persistence may be measured by the magnitude of estimated fractional differencing parameter. The validity of purchasing power parity model can be tested by ARFIMA models that are applied on real exchange rate. The issues of the inflation persistence magnitude and the purchasing power parity model in the CSEE region have been recently discussed under a different econometric set-up in Mladenović and Nojković 2012 and Mladenović 2012.

In the rest of paper the same CSEE region is considered with the purpose of analyzing daily stock market returns within the ARFIMA and FIGARCH set-up.

4. Preliminary Results of Empirical Modeling

The presence of long-memory both in the level and variability of data are considered within the two-pass approach. Its use has been advocated by Kasman et al. 2009. In our empirical work another set of diagnostic tools and model specifications were chosen. Also, instead of estimating both fractionally differencing parameters at the same time, we first estimated ARFIMA models for the returns level and then, based on the residuals derived, apply FIGARCH models.

In the first step long-range dependence in the level of returns is considered by calculating tests that are reviewed in section 3.

Table 2: Testing for long-memory in the level of stock exchange indices daily returns

Return	Q(20)	Hurst-Mandelbrot R/S	Lo R/S for lags		The runs Test	GPH estimate
			5	10		
BUX	79.97 (0.00)	1.41	1.39	1.41	0.18 (0.85)	0.005 (0.80)
PX	73.57 (0.00)	1.63	1.65	1.66	-1.12 (0.26)	-0.02 (0.23)
BELEX15	340.64 (0.00)	3.00	2.29	2.17	-7.84 (0.00)	0.23 (0.00)
CROBEX	72.18 (0.00)	2.03	1.88	1.81	-2.03 (0.04)	0.06 (0.00)
SOFIX	224.77 (0.00)	3.17	2.58	2.41	-4.84 (0.00)	0.16 (0.00)

Note: p values of corresponding test statistics are given in parentheses. The 95% confidence interval for both R/S statistics is (0.81,1.86). Q represents the Box-Ljung statistics for testing autocorrelation of order 20 in the data. GPH estimate is obtained for the sample of size T/2.

All stock market returns exhibit high autocorrelation. However, R/S statistics of Hurst-Mandelbrot and Lo type suggest that long-range dependence exists in three time series: returns of BELEX15, CROBEX and SOFIX. The results are confirmed by the runs test and significance of GPH estimate of fractional difference parameter. Based on these results we estimated ARFIMA type models as a final phase of our first step.

Within the second step of our approach we calculated the same statistics of long-range dependence but now on the absolute values of residuals from estimated ARFIMA models. In this way we investigate whether long-memory appears in volatility. Results given in Table 3 clearly indicate that this is the case for all data, thus highlighting the relevance of FIGARCH framework. Therefore, in the final phase of the second step we estimated FIGARCH models for five returns.

Table 3: Testing for long-memory in the variability of daily returns (Absolute values of residuals from ARFIMA models)

	Hurst-Mandelbrot R/S	Lo R/S for lags		The runs test	GPH estimate
		5	10		
BUX	5.70	3.93	3.22	-4.37 (0.00)	0.21 (0.02)
PX	5.97	3.78	3.05	-6.66 (0.00)	0.29 (0.02)
BELEX15	5.96	3.70	3.13	-9.80 (0.00)	0.31 (0.00)
CROBEX	6.40	4.05	3.39	-8.56 (0.00)	0.27 (0.00)
SOFIX	5.52	3.30	2.70	-13.17 (0.00)	0.35 (0.00)

5. Estimated Models

For each stock market return estimated model is provided. Specifications with the best statistical performances are reported. All FIGARCH specifications assume an error term that follows t-distribution to account for heavy tails in empirical distributions of returns. Additionally, to take care of possible asymmetric reaction of volatility to shocks of different signs, asymmetric power specification has been reached in two cases.

Table 4: Modelling return on BUX

Mean equation		
No variables		
Volatility equation		
Parameter	Estimate	t-ratio
α_0	0.295	4.21
d	0.281	6.46
Φ_1	-0.194	-4.23
Φ_2	-0.058	-1.72
γ_1	0.300	4.32
δ	1.863	14.23
v	10.603	5.95
Residual diagnostics		
Q(10)=20.98(0.02), Q(20)=29.79(0.07) Q^2(10)=7.43(0.49), Q^2(20)=18.67(0.41) Skewness=-0.02(0.65), Excess kurtosis =0.72(0.00) Engle-Ng test =2.20(0.53) Nyblom stability test= 1.15 Adjusted Pearson χ^2 goodness-of-fit test: Observations 40: p-value=0.40 Observations 60: p-value=0.62		

Note to Tables 4-8: AR(p) stands for an estimate of autoregressive parameter of order p. Q denotes the Box-Ljung statistic for testing the autocorrelation of order 10 or 20 in the level of standardized residuals. Q^2 refers to the Box-Ljung statistic for testing the autocorrelation of order 10 and 20 in squared standardized residuals. Engle-Ng test is reported for jointly investigating the presence of sign bias, negative size bias and positive size bias in standardized residuals. The adjusted Pearson χ^2 goodness-of-fit test compares the empirical distribution of standardized residuals with the theoretical one. Corresponding p-values are associated with the number of degrees of freedom that is equal to the number of cells minus number of estimated parameters plus one. Nyblom stability test refers to joint stability of all parameters. Its critical values depend on the number of parameters (Hansen 1992; Nyblom 1989). The 5% critical value is not exceeded in any model estimated.

Table 5: Modelling return on PX

Mean equation		
Parameter	Estimate	t-ratio
d	0.067	2.30
AR(1)	-0.012	-0.35
AR(2)	-0.105	-4.66
AR(3)	-0.071	-3.28
Volatility equation		
Parameter	Estimate	t-ratio
α_0	0.127	3.71
d	0.421	4.86
Φ_1	0.173	2.70

β_1	0.482	4.41
γ_1	0.402	4.87
δ	1.477	10.04
v	8.817	7.34
Residual diagnostics		
Q(10)=15.60(0.10), Q(20)=21.20(0.39) Q^2(10)=8.89(0.35), Q^2(20)=28.55(0.05) Skewness=-0.24(0.00), Excess kurtosis =1.25(0.00) Engle-Ng test =7.47(0.06) Nyblom stability test= 2.31 Adjusted Pearson χ^2 goodness-of-fit test: Observations 40: p-value=0.21 Observations 60: p-value=0.12		

Table 6: Modelling return on BELEX15

Mean equation		
Parameter	Estimate	t-ratio
d	0.384	6.65
AR(1)	-0.091	-1.47
AR(2)	-0.100	-2.43
AR(3)	-0.205	-5.76
AR(4)	-0.092	-2.40
AR(5)	-0.121	-3.52
AR(6)	-0.108	-3.11
AR(7)	-0.078	-2.37
AR(8)	-0.067	-2.16
AR(9)	-0.045	-1.51
AR(10)	-0.080	-2.92
AR(11)	-0.079	-2.85
AR(12)	-0.049	-1.79
Volatility equation		
Parameter	Estimate	t-ratio
α_0	0.152	2.25
d	0.496	7.55
Φ_1	-0.173	-2.380
v	4.80	9.33
Residual diagnostics		
Q(10)=15.56(0.11), Q(20)=24.06(0.24) Q^2(10)=3.03(0.96), Q^2(20)=6.17(0.99) Skewness=0.26(0.00), Excess kurtosis =6.74(0.00) Engle-Ng test=1.38(0.71) Nyblom stability test = 1.36 Adjusted Pearson χ^2 goodness-of-fit test: Observations 40: p-value=0.13 Observations 60: p-value=0.11		

Table 7: Modelling return on CROBEX

Mean equation		
Parameter	Estimate	t-ratio
d	0.106	4.04
AR(1)	-0.053	-1.69
AR(2)	-0.064	-2.83
Volatility equation		
Parameter	Estimate	t-ratio
α_0	0.152	3.25
d	0.379	9.44
Φ_1	-0.164	-3.21
v	4.30	12.78
Residual diagnostics		
Q(10)=17.86(0.06), Q(20)=22.66(0.31) Q^2(10)=7.79(0.56), Q^2(20)=9.65(0.96) Skewness=-0.2(0.00), Excess kurtosis =10.75(0.00) Engle-Ng test =1.18(0.76) Nyblom stability test= 1.73 Adjusted Pearson χ^2 goodness-of-fit test: Observations 40: p-value=0.41 Observations 60: p-value=0.57		

Table 8: Modelling return on SOFIX

Mean equation		
Parameter	Estimate	t-ratio
d	0.250	6.65
AR(1)	-0.141	-2.74
AR(2)	-0.033	-0.92
AR(3)	-0.061	-2.18
AR(4)	-0.013	-0.52
AR(5)	-0.082	-3.55
AR(6)	-0.031	-1.31
AR(7)	-0.064	-2.87
AR(8)	-0.003	-0.13
AR(9)	-0.039	-1.88
Volatility equation		
Parameter	Estimate	t-ratio
α_0	0.095	2.91
d	0.528	13.83
Φ_1	-0.203	-4.37
v	5.00	12.91

Residual diagnostics
Q(10)=14.83(0.14), Q(20)=29.87(0.07) $Q^2(10)$=8.57(0.48), $Q^2(20)$=13.26(0.82) Skewness=-0.07(0.11), Excess kurtosis =2.7(0.00) Engle-Ng test=1.38(0.67) Nyblom stability test= 2.45 Adjusted Pearson χ^2 goodness-of-fit test: Observations 40: p-value=0.11 Observations 60: p-value=0.09

Results obtained are summarized in Table 9.

Table 9: Summary results of modelling returns

BUX	PX	BELEX15	CROBEX	SOFIX
Mean equation				
ARFIMA (0,0,0)	ARFIMA (3,0.07,0)	ARFIMA (12,0.38,0)	ARFIMA (2,0.11,0)	ARFIMA (9,0.25,0)
Volatility equation				
AP-FIGARCH (*0,0.28,2*)	AP-FIGARCH (*1,0.42,1*)	FIGARCH (*0,0.5,1*)	FIGARCH (*0,0.38,1*)	FIGARCH (*0,0.53,1*)
Estimated number of degrees of freedom of t-distribution				
10	8	4	4	5

The level of five market returns series are characterized by different correlation patterns. For modelling returns on BUX indices ARFIMA specification was not needed. Similar behaviour was discovered for PX returns since the fractional difference parameter was estimated to be 0.07 while three lags of autoregressive structure were included. Much higher long-range dependence was detected for the rest of the sample, since estimated parameter of fractional differencing takes values 0.11, 0.25 and 0.38 respectively for returns on CROBEX; SOFIX and BELEX15. For adequate modelling of BELEX15 and SOFIX returns substantially high number of autoregressive components was included.

Long-memory was found in volatility of all five returns suggesting that volatility exhibits long-lasting temporal dependence. It is characterized by the fractional differencing parameter 0.28 and 0.42 for BUX and PX returns respectively. Asymmetric reaction of volatility to sign of the shock was recognized, while estimated number of degrees of freedom of t-distribution is 10 and 8 respectively. Less sophisticated specification was found for the rest of the sample. Namely, simple ARCH(1) structure under fractionally-differenced volatility was estimated for CROBEX, SOFIX and BELEX15 returns. Estimates on fractional differencing parameter are of higher magnitude than in the previous two cases: around 0.5 from BELEX and SOFIX and 0.38 for CROBEX. Estimated numbers of degrees of freedom of t-distribution are also similar, since they take integer values 4 and 5.

Comparative analysis implies that considered markets differ significantly. We may argue that financial markets in Hungary and Czech Republic are more resistant to speculative attacks, contrary to markets in Croatia, Serbia and Bulgaria. However, uncertainty on risk movement appears at relatively high level in all five capital markets, but it is more pronounced in Croatia, Serbia and Bulgaria.

From econometric point of view, ARFIMA and FIGARCH models capture time-series properties satisfactory well, which stresses out their relevance in describing dynamics of stock market returns from CSEE region. These models outperform specifications that have been previously used, as in Mladenović et al. 2012.

6. Conclusions

The paper investigates the long-memory in mean and volatility of five capital markets returns from Central and South-Eastern Europe. It is found that long-memory does not exist in returns on BUX (Hungary) and PX (the Czech Republic). However, long-range dependence has been estimated as highly significant in returns of SOFIX (Bulgaria), CROBEX (Croatia) and BELEX15 (Serbia). Thus, ARFIMA specifications were proven as appropriate modelling framework.

There is a strong evidence that long-memory plays a key role in describing conditional variability. This holds for all data considered. Thus, FIGARCH specification was chosen in all five cases as an appropriate econometric set-up to model slowly diminishing long-range dependence of volatility. Significant asymmetric behaviour of fractionally-differencing volatility has been revealed for BUX and PX returns, implying that volatility also depends on the sign of unanticipated random shock. Namely, negative shocks have influence of larger magnitude than positive ones.

Previous comprehensive analysis on this topic was conducted for the sample that covers 1992 (1997) - 2006 period and more time series than in our paper (Kasman et al. 2009). Serbia was not considered due to its late start of financial market development. Our results differ to some extent to the findings reported in Kasman et al. 2009. This is to be expected given that our data consist of market returns for the period 2001 (2005) – 2012, thus incorporating the effect of the 2008 global crisis. Long-memory of the stock market returns was estimated to be of similar level only for the Czech data. We found it to be of smaller magnitude for the Hungarian data and substantially greater in Croatia and Bulgaria. The last two economies appear to be more sensitive to external shocks, while Hungarian stock market exhibits higher level of development and stability. Our results on volatility dynamics suggest that Croatian and Bulgarian market indices display high level of persistence that was observed in the previous study as well. On the other side, long-memory in volatility of Hungarian and Czech data was also confirmed, but with the pattern showing reduction of persistence over time.

The finding of long-memory presence in Bulgarian, Croatian and Serbian stock returns rejects weak-form market efficiency hypothesis. It implies that speculative earnings could be systematically obtained. Although financial markets in Hungary and Czech Republic share the same feature, performances towards reaching more efficiency have been detected in these two economies.

The long-range dependence in volatility of all five returns indicates that uncertainty over time-series fluctuations decays extremely slowly. This highlights the relevance of employing FIGARCH specification to estimate dynamics uncertainty, which is important in the risk management analysis.

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